THE HIGH VOLTAGE CORONA IN AIR.

By J. B. WHITEHEAD.

(*Read April 21, 1911.*)

The term "corona" as employed by electrical engineers refers to the luminous envelope which surrounds a bare electrical conductor when its potential is raised above a certain value. As the voltage of long-distance transmission lines has been raised to higher and higher values in order to reduce the size and cost of the conductors and so increase the distance of economical transmission, a limiting condition has been found in the insulating properties of the atmosphere. For each definite space separation and size of conductors, above a certain value of voltage the regions immediately surrounding the conductors become luminous, and a power loss sets in which increases rapidly with further increase in voltage.

These facts were first noted by electrical engineers in this country in 1896. It was promptly recognized that the region in the immediate neighborhood of the conductors is subject to the greatest electric intensity and that the phenomena are due to local though restricted break-down of the air. This was corroborated not only by the presence of the luminous envelope immediately around the conductors, for voltages above that at which the loss begins, but by study of the effect of changing the size and separation of conductors; decreasing separation and size both increase the surface electric intensity and therefore lower the voltage at which loss begins. The electric intensity at the surface of the conductor may be readily calculated in most cases that occur from the voltage and from the separation and sizes of the conductors. It is directly proportional to the voltage in all cases. The term "corona" was first used by Steinmetz in 1898 to describe the luminous envelope and has been generally adopted by engineers.

Many measurements have been made on electric power transmission lines in efforts to determine the law connecting the voltage at which loss begins with the physical constants of the line. These measurements have shown marked inconsistencies among themselves, the results on the same lines on different days being often at variance. A number of laboratory investigations, in which the widely varying conditions of a transmission line are under control, have naturally followed. They have indicated with a rather wide variation in numerical values that the critical voltage or voltage at which corona begins on round wires varies inversely as the temperature and directly as the pressure; also that the electric intensity under which the air near the surface of the conductor breaks down has not a constant value but increases markedly for conductors of small diameter; and further that the value of the intensity at which break-down begins is that corresponding to the maximum value of the alternating wave, and is independent of the material of the conductor.

The general nature of the influence of temperature and pressure could probably have been predicted from numerous investigations of the discharge of electricity through gases; the quantitative relations for pressures near that of the atmosphere do not, however, appear to have attracted the physicist, nor indeed have they as yet been satisfactorily determined for the voltage of corona formation by experimental engineers. The accumulated results of physical investigation and theory, however, offer no obvious explanation of the rise of the critical surface intensity for smaller wires, nor of the influences of the form and frequency of alternating voltage. The fact that the corona voltage is that corresponding to the maximum value of the alternating wave has been proven by stroboscopic methods and by the use of distorted wave shapes. It indicates that the time element involved in the process of break-down of the air is short compared with the periods of the common alternating current circuits. The apparent sharpness of the connection removes many objections to the use of the alternating electromotive force as a means of investigation, and renders available its many advantages. It is only necessary to know the shape of the alternating wave and this may be obtained readily by several well known methods. Effective values as read on direct reading instruments may thus be used and the corrective factor for the maximum value is obtained from the shape of the wave.

Many of the inconsistencies among the measurements on existing transmission lines and those made in laboratories arise in the difficulties of measuring the power in high voltage circuits; the instruments must be placed in the low voltage side of transforming apparatus, the losses in which, being generally greater than those to be measured, introduct a troublesome source of error. The appearance of the visible corona has been used by laboratory workers as an indication of the beginning of loss through the air. With proper precautions this method may be very reliable but its use is generally attended by danger of subjective and other error. As a result of the discrepancies among these approximate determinations of various investigators, there has appeared much speculative suggestion of the presence of other unrecognized influences, as for example the moisture content of the air, the presence of "free" or natural ionization, an abnormal property of air when near a small wire, etc.

The present problem therefore resolves itself into two parts: first, a satisfactory method for the determination of the law under which the air in the neighborhood of a long straight and usually cylindrical conductor breaks down under electric strain; and second, the law governing the amount of loss when the voltage is carried above the critical value. A year ago the writer[1] described a method by which it is possible to determine the voltage at which the air breaks down near a round wire to a maximum inaccuracy of a few tenths of one per cent. The original paper may be consulted for the details, but the principle is simple and may be described briefly. The wire is stretched along the axis of a metal cylinder and the voltage is applied between them. Air may be passed through the cylinder by means of two lateral tubes near the ends, the walls of the cylinder at these points being drilled with a number of small holes. Close to one set of these holes and outside the cylinder a wire mesh electrode connected to a sensitive electroscope is placed. As soon as the air around the wire breaks down under increasing voltage, copious ionization sets in which causes a rapid leak from the

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[1] J. B. Whitehead, *Proc. A.I.E.E.*, p. 1059, July, 1910.

charged electroscope. The initial discharge of the electroscope is very sharply marked. Observations may be repeated at will and after any interval; when corrected for temperature and pressure a most satisfactory constancy of results is obtained. Fig. 1 indicates the essential parts of the apparatus. In the following a short description is given of the results of investigations of the influence of the diameter of the conductor, of stranding the conductor, of the alternating frequency, of wave form, of pressure, of moisture content and of temperature on the electric intensity at which atmospheric air breaks down. The experiments on temperature, moisture content and diameter of conductor are given in the paper mentioned above. The results of the remaining investigations are first given here. No attempt is made to describe the details of the experiments. For these the reader may refer to the earlier paper and also to one shortly to be presented to the American Institute of Electrical Engineers in which the practical bearing of the results will be discussed.

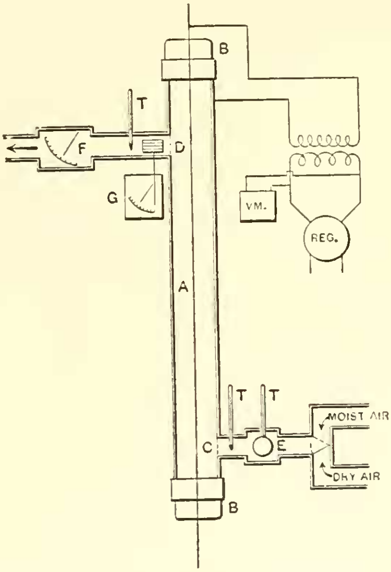

Fig. 1. Arrangement of apparatus.

Influence of Diameter of Conductor.—For convenience of reference a condensed table of the results on this portion of the work is given in Table I. and the results are plotted graphically in Fig. 2. For comparison, points observed by other investigators are also shown. The values are all corrected for temperature, pressure and wave form and give the maximum values of the electric intensity at which the air breaks down under a pressure of 760 mm. of mercury, and at temperature 21° C. I am indebted to Professor Alexander Russell for pointing out that these results obey a very simple law. If E be the critical electric intensity in kilovolts per centimeter and d the diameter of the conductor in centimeters, the curve of Fig. 2 obeys closely the equation:

TABLE I.

Relation Between Diameter and Critical Surface Intensity.

Diameter, cm.	Material of Wire.	Diameter of Tube, cm.	Material of Tube.	Critical Primary Volts Corrected.	Ratio.	Maximum Critical Surface Intensity.
0.089	Copper	4·9	Brass	74.5	125.09	77,100
0.122	“	“	“	44.0	250.18	70,950
“.	“	“	“	87.8	125.09	70,800
0.156	“	“	“	97·0	“	65,880
“	“	“	“	97·0	“	55,880
0·205	“	“	“	109.5	“	61,350
“	“	“	“	55·0	250.18	61,500
“	“	6.35	“	60.5	“	62,080
“	“	“	“	60.2	“	61,780
“	“	“	“	(8) 59·9	“	61,680
0.254	Aluminum	“	“	65.9	“	58,750
0.276	Copper	“	“	68·9	“	58,080
“	“	“	“	68.8	“	
“	“	“	“	68.5	“	
“	“	“	“	69.0	“	
“	“	“	“	(8) 68.9	“	58,080
“	“	9.52	Steel	77·3	“	57,650
0·325	Aluminum	6.35	Brass	72.8	“	55,000
0.347	Copper	6.35	“	75·13	“	54,500
0 3405	“	9.52	Steel	85.7	“	55.100
0.399	Steel	“	“	92.5	“	53,050
0.475	“	“	“	100.6	“	51,400

$$E = 32 + 13.4\ 1/\sqrt{d}. \qquad (1)$$

The observed values of Table I. are compared with the values calculated from the above formula in Table II. The percentage error is also given, and it is seen that with one exception the difference is well within one per cent. The exception refers to an aluminum wire which could not be polished to a clean surface; a rough sur-

TABLE II.

Diameter, cm.	Kilovolts per Centimeter.		Difference, Per Cent.
	Calculated.	Observed.	
.089	76,950	77,100	+ 0.19
.122	70,400	70,875	+ .67
.156	65,950	65,880	— .1
.205	61,600	61,680	+ .13
.254	58,600	58,750	+ .25
.276	57,500	58,000	+ .87
.325	55,600	55,000	— 1.08
.340	54,980	55,100	+ .21
.347	54,780	54,500	— .51
.399	53,230	53,050	— .23
.475	51,460	51,400	— .11

face invariably lowers the critical intensity. The corresponding point falls below the curve of Fig. 2.

The closeness with which this simple law is followed by the measurements suggest a considerable value of the method for a

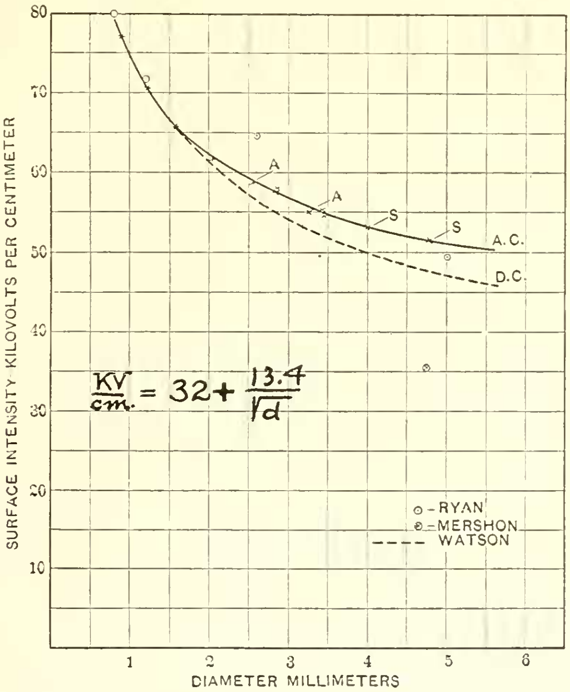

FIG. 2. Relation of critical intensity and diameter.

physical investigation of the nature of the process involved in the electrical break down of the air. The formula indicates that the value of electric intensity of a uniform field or near the surface of a plane conductor, at which air would break down is 32 kilovolts per centimeter. The work of von Seweidler, Townsend and others indicates that at about 30 kilovolts per centimeter, secondary ionization or ionization by collision sets in between parallel plates subjected to a difference of potential. In a later paragraph several other experiments are described which indicate that the start of the corona in air is due to secondary ionization. So far as the writer is aware however no theory has been advanced to explain the influence of the curvature of the conductor. That the nature of the molecular structure of the air is concerned there can be no doubt, but the variation of values of critical intensity occurs within a range of diameter many orders of magnitude greater than molecular dimensions, and is related to the diameter in a way which offers no suggestion of explanation.

Effect of Stranding the Conductor.—It is quite obvious that if the surface intensity is the determining factor in the voltage at which corona occurs, then a stranded conductor should have a critical voltage lower than that of a solid conductor of a diameter equal to that of a circle tangent to the strands. On the other hand it is not obvious that the critical voltage of a stranded conductor would be less than that of a solid conductor of equivalent cross section, for the diameter of the latter will always be less than that of the enclosing circle of the former. Evidently also the relations will vary with the number of strands. The question is of importance since all of the larger transmission lines consist of cables or stranded conductors.

A series of observations was made on a number of cables of stranding ranging from three to nine conductors uniformly filling the outer layer. The interior space was filled with a single wire or several wires of suitable size, but in each conductor the wires of the outer layer were all of the same size, .162 cm. diameter. The cables were clean and smooth and drawn tight along the central axis of the outer cylinder of the apparatus. The results are condensed in Table III, in which comparison is made between the diameter of a solid

TABLE III.

Strands.	(a)	(b)	(c)	(b/c)	(c/a)	Spiral Pitch, Diams.
3	.349	.272	.247	1.10	.708	10.9
4	.404	.332	.32	1.04	.792	8.65
5	.45	.381	.37	1.03	.822	9.89
6	.49	.430	.42	1.026	.857	12.3
7	.541	.48	.465	1.032	.868	12.3
8	.589	.53	.516	1.027	.877	10.78
9	.64	.581	.567	1.025	.886	10.9
3	.336	.27	.207	1.305	.616	none
4	.378	.312	.25	1.248	.665	none

circular conductor having the same critical voltage as that observed for the cable (column *c*) and the diameters of the circle just enclosing the cable (*a*) and that given its equivalent section (*b*). The ratios b/c and c/a are given in the last two columns, and are plotted in their relation to the number of strands in Figs. 3 and 4 respectively. Fig. 3 indicates that if instead of a solid conductor a stranded conductor of equivalent cross section be used the critical voltage will be lowered, but by less than three per cent. if the number of strands is greater than 5. For a three-strand cable the lowering is ten per cent.

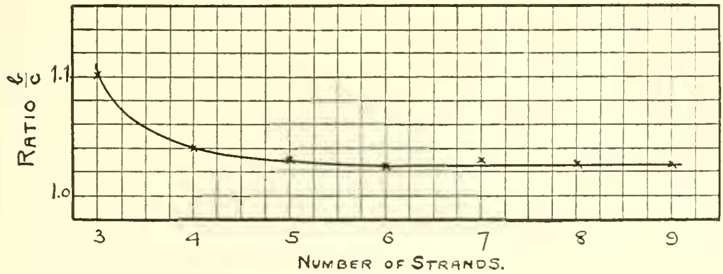

Fig. 3.

The ratio c/a is more important however. This ratio compares the diameter of a solid conductor having the same critical voltage as the cable, with the actual overall diameter of the cable. It therefore refers the behavior of a given cable, with regards critical voltage, to a solid round wire whose diameter is expressed as a fraction of the overall diameter of the cable. This is a more logical basis of comparison than the other since the interior of a multistrand cable may be made up in such manner as to cause a considerable variation in its cross section. In fact many transmission cables have centers of hemp, or other material, the entire conducting section residing in the single outer layer of strands. Thus Fig. 4 shows that a three-strand cable has a critical voltage which is that of a single wire of seven tenths the overall diameter of the cable. At nine strands the equivalent single diameter is still less than .9 that of the cable.

In a stranded conductor the strands are always spiralled. The pitch of the spiral for the cables described above is given in Table III. The spiral arrangement of the strands tends to lessen the value of the electric intensity on the outer surfaces of the strands since the equipotential surfaces are rendered more nearly cylindrical about the axis of the cable. The values of maximum surface electric intensity for cables of various numbers of strands and in which there is no spiral may be computed from an expression given by Jona[2] and due to Levi-Civita. This expression involves a hypergeometrical series whose evaluation requires some labor. As it makes no allowance for the spiralling of the strands no deduction may be drawn from the present observations as to the actual intensity at which corona occurs on the stranded conductor. Values deduced from the expression should, however, be of great value in the study of the nature of the breakdown of the air when taken in conjunction with measurements on cables without spirals. For in these cases the maximum electric intensity at the outer edge of a strand would obtain over a narrow circumferential distance, while the same intensity reached at the surface of a single wire obtains over a whole circumference. A comparison of corona voltages in the two cases should throw light on the distances involved in the process of secondary ionization and kindred phenomena.

At the bottom of Table III. there are given the results of observations on a three- and a four-strand conductor in which there was no spiral. The size of the strands was the same as that of the foregoing cables. The strands were carefully straightened, polished,

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[2] Jona, *Trans. Int. Elect. Congress, St. Louis,* 1904, Vol. II., p. 550.

and built up by soldering with a fine blow flame so that the strands were uniformly tangent to each other throughout. The results indicate the further lowering of the critical voltage when spiralling is absent. The ratio c/a falls from .71 for the spiralled three-strand to .61, and the difference for the four-strand is somewhat greater. The pitch of the spirals of the cables investigated does not appear to follow any regular rule. This irregularity however does not appear to have any corresponding effect on the points of the curve

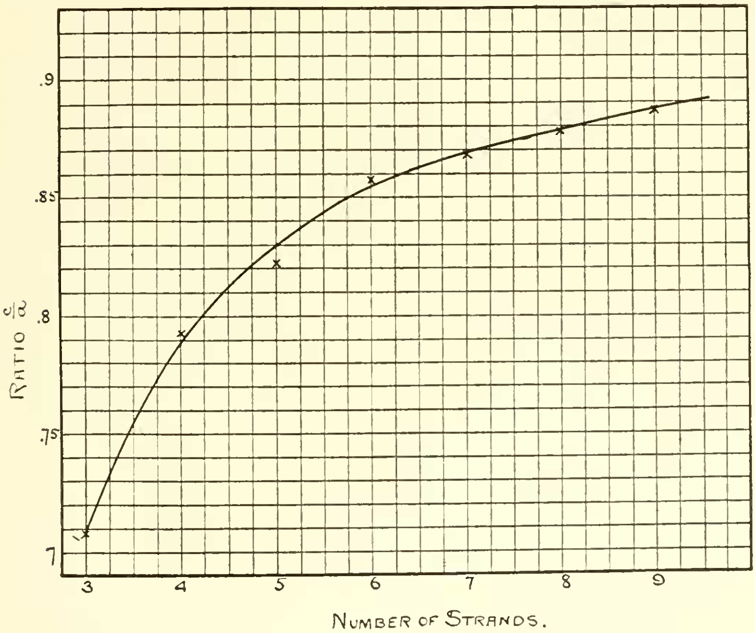

Fig. 4.

of Fig. 4. From this it may be concluded that for a pitch of spiral less than twelve diameters there is no gain on the ground of lessened surface intensity due to the more uniform distribution of the electric field.

At this writing the author has been unable to obtain solutions of Levi-Civita's expression as applied to three and four strands. These would permit by the foregoing results a knowledge as to how the maximum corona intensity for a round wire compares with that at the surface of the same wire when made up into a three- or four-strand cable without spiral.

Influence of Frequency and Wave Form.—By the use of a cathode ray oscillograph in the high voltage circuit Ryan in 1904 showed that the appearance of corona was accompanied by a hump or peak on the charging current wave in the neighborhood of the maximum of voltage. The writer by stroboscopic methods has shown that the corona is periodic, appearing every half cycle and that its first appearance with rising voltage coincides accurately with the maximum of the voltage wave. Also the duration of the corona, with steady circuit conditions, may be reduced with lessening voltage to a very small fraction of the period of the alternating electromotive force. Thus a corona which was found to exist for only one twentieth of a period at the crest of the voltage wave of a 60-cycle circuit was plainly visible in a darkened room. It is evident, therefore, that the interval of time involved in corona formation and cessation is extremely short. For these reasons it has been supposed that the appearance of corona depends only on the maximum value of voltage occurring in the cycle, and is therefore independent of the frequency. Experience with existing lines indicates that if there is an influence of frequency it is small for the range between 25 and 60 cycles. The closeness with which the critical voltage may be read by the method described gave promise of discovering any comparatively small differences due to variation of frequency. Several series of tests were therefore made with different sizes of wire. The observations are not recorded here as the points on the curve of Fig. 5 are a sufficient indication of their accuracy. The range from 15 to 90 cycles was obtained from two generators, and the voltage from a 10-KW. 25-cycle 100,000-volt transformer. The transformer had also a low voltage secondary coil. On the curves the values of voltage are those measured at the terminals of this coil; these values are therefore proportional to the voltage in the high tension winding and therefore to the electric intensity at the surface of the wire. These observations were made with rods .716 cm. and .635 cm. in diameter placed at the center of a pipe 120 cm. long and 30 cm. in diameter. The observations were taken as a continuous set, interruption being necessary for only a few seconds to change generators. There were consequently no appreciable variations in temperature or pressure.

The results as taken are plotted in the lower curves of Fig. 5 in which observations for ascending and descending values of frequency are plotted as crosses and circles respectively. The irregular shape of these curves repeated itself accurately in experiments over the same range of frequency with other wires. Since the transformer was operating over a wide range of frequency at approximately the same value of voltage, and its magnetizing current was therefore variable, a variation of wave form due to the armature reaction of the generator appeared probable. Oscillograms were therefore taken of the voltage at the terminals of the low voltage secondary coil at frequencies 20, 35, 55, 60, 65 and 91 cycles, and at transformer excitation corresponding to 50 volts on the same coil. The ratios of maximum to effective values of these waves were

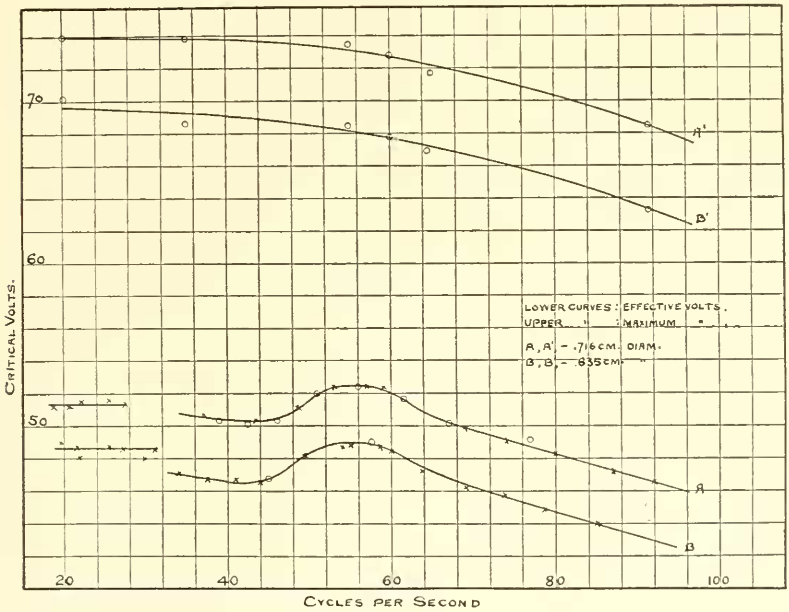

Fig. 5.

then determined by micrometer measurements of ordinates taken every 7.5 degrees over two half waves. The several values of this ratio so obtained revealed a minimum at 55 cycles thus explaining the rise in the lower curves of Fig. 5 at that frequency. In the upper curves the points indicated are the voltage of the lower curve multiplied by the ratio of maximum to effective value as calculated from measurements of the oscillograms for the corresponding frequencies.

The upper corrected curves of Fig. 5 show a lowering of the critical voltage with increasing frequency. The result leaves something to be desired in the accuracy of location of the points upon the curve. It should be noted however that owing to the magnification of the scale, the error of the points off the upper curves and the 25-cycle portion of the lower curves is only about 1 per cent. Several other sets of observations for different sizes of wire reveal curves of the same general characteristics. The measurement of the ratio of maximum to effective value from an oscillogram is subject to considerable error. The maximum at 55 cycles, however, on the lower curves is brought below the values for lower frequencies when the correcting factor is introduced, and particularly, the lowering at 91 cycles is far too great to be questioned on the score of a possible error of this nature. The curves therefore show with a fair accuracy the nature of the variation of the critical voltage with the frequency. This variation within the range of the present commercial frequencies 25 to 60 cycles per second, is only about 2 per cent.

Influnce of Pressure.—The influence of pressure on the various forms of spark discharge has been closely studied. Paschen's[3] law states that the sparking potential for a given spark length is directly proportional to the pressure; his investigations covered the range of pressure between 10 and 75 cm. of mercury. Carr[4] has shown that this linear relation extends down to pressures of a few millimeters if the spark lengths are not greater than 1 cm. but does not obtain for lower pressures. Townsend[5] has shown that the potential

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[3] Paschen, *Wied. Ann.*, XXXVII., 79, 1889.

[4] Carr, *Proc. Roy. Soc.*, LXXI., 374, 1903.

[5] Townsend, *Phil. Mag.*, VI., 1, 198, 1901.

gradient at which secondary ionization sets in when electricity is passing through a gas is directly proportional to the pressure. Watson[6] investigated the spark length between spheres up to fifteen atmospheres and found that the spark potential increases with the pressure in an approximately linear relation. From the general similarity between the corona and the brush form of spark discharge, therefore, a linear relation between pressure and critical surface intensity, or the potential gradient at which corona begins is to be expected. Apparently the only study of the influence of pressure on the formation of the alternating corona is a single set of observations by Ryan[7] on a wire .32 cm. in diameter placed at the center of a cylinder 22.2 cm. in diameter. He observed the alternating voltage at which the visible corona appeared for the range of pressure between 45 and 90 cm. of mercury; the alternating frequency was 130. The resulting linear relation is given as between the kilovolts K actually applied and the pressure in inches of mercury, $K = 2.93 + .902\,b$.

In Table IV. are given the results of a typical series of observations on the influence of pressure on corona voltage; the values are those for a wire .152 cm. in diameter. The wires were clean and straight and centered accurately on the axis of the outer cylinder of the apparatus which has been briefly described. This cylinder has a diameter of 9.52 cm. The ends were closed with ebonite caps of the same diameter and 5 cm. deep. The side tubes were also closed by caps, and the leading-in wire to the discharge electrode passed through a column of sulphur supported in hard rubber; no troubles with either insulation or air leak were encountered with this arrangement. All joints were sealed with a mixture of bees wax and resin and pressures between 30 and 100 cm. of mercury were reached without trouble. The discharge electrode was placed inside the upper side tube and within one or two millimeters of the grating formed by the holes drilled in the outer cylinder; in the earlier work it was found that a flow of air from the cylinder over the electrode contributed little to the sharpness with which the condition of

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[6] Watson, *Electrician*, 62, 851, 1909.

[7] Ryan, *Proc. A.I.E.E.*, XXIII., 101, 1904.

breakdown was indicated, the initial discharge of the electroscope occurring at the same value for both moving and stationary air. The results of Table IV. are plotted in the lower line of Fig. 6

TABLE IV.

Crit. Prin. Volts. Ratio, 1 : 125.			Manometer.			Pressure, mm.
			Right.	Left.	Diff.	
102.2	102.2	102.2	487.5	587.5	—100	659.5
97.5	97.5	97.2	459.5	605.5	146	613.5
91.3	91.3	91.2	427	628.5	201.5	558
87.2	87.5	87.8	407.5	642	234.5	525
83	83.2	83.4	386.5	656	269.5	490
79.9	80	80	367.5	669 5	302	457.5
80.5	80.7	80.6	371	666.5	295.5	464
74	74	74	340	688.5	348.5	411
68.1	68.1	68.1	313.5	707	393.5	366
94.2	94.2	94.2	439	617	178	581.5
106.5	106	106.2	499	576.5	77.5	682
114.5	114.9	114.8	545.5	545.5	0	759.5
Ratio 1 : 250.						
57.5	57.4	57.5	545.5	545.5	0	759.5
59.8	59.9	59.8	570.5	530.5	+ 40	799.5
61.8	61.6	61.7	592	516.5	75.5	835
64	64	63.8	618.5	499.5	119	878.5
66			641	486	155	914.5
67.7	67.7	67.7	661	473.5	197.5	957
69.8	70	69.9	687.5	457	230.5	989.5
71.6	71.6	71.7	710	444	266	1025.5

between the values of voltage at the primary terminals of the transformer and the pressure in millimeters of mercury. This voltage is directly proportional to the corresponding value of potential gradient at the surface of the wire. The ratios of transformation were 1 to 125 and 1 to 250, the frequency 60, and the ratio of the maximum to the effective value of the alternating wave of electromotive force, as measured from an oscillogram as already described, was 1.46. The temperature was 24° C. The results for a .276-cm. wire are also plotted in Fig. 6. The equations of the lines as drawn in Fig. 6 have no significance since they apply to a particular combination of wire and outer cylinder. The values of surface potential gradient have therefore been calculated from the expression:

$$\frac{dV}{dr} = \frac{E}{r \log \frac{R}{r}}, \qquad (2)$$

in which E is the maximum value of the potential difference between wire of radius r and outer cylinder of radius R, and which in this case is the effective voltage multiplied by 1.46. Expressed in terms of electric intensity at which corona begins, in kilovolts per centimeter, and pressure in centimeters of mercury, the equation for the .152 cm. wire is:

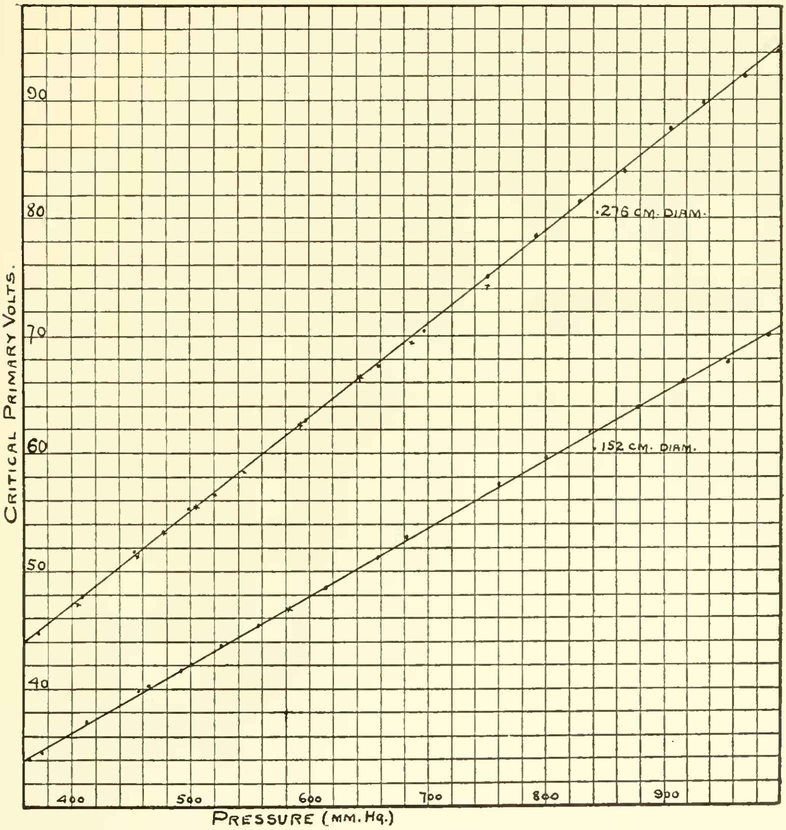

Fig. 6.

$$\frac{d(KV)}{dr} = 15.2 + .673\,p, \qquad (3)$$

and for the .276 wire:

$$\frac{d(KV)}{dr} = 11.6 + .595\,p, \qquad (4)$$

While both equations are linear it is seen that the slope of that for the smaller wire is the steeper, that is that the variation of the critical surface intensity with the pressure is greater the smaller the wire. It is interesting to note that the values at 76 cm. pressure 66.2 and 57 correspond extremely closely with the values 66.4 and 57.7 observed a year before and so calculated from the equation of Fig. 2.

If Ryan's results for a .317-cm. wire be expressed in the same terms used in the above formulae, the resulting equation of the line is:

$$\frac{d(KV)}{dr} = 6.15 + .744\,p. \qquad (5)$$

The slope of this line is greater than that of either the .152-cm. or the .276-cm. wire as expressed in equations (3) and (4), although the larger size of wire should cause the slope to be less; also the initial constant term is considerably less; further the value of critical surface intensity at 76 cm. pressure indicated by formula (5) is 62.6, while that calculated from formula (1) and therefore frequently observed by the writer is 55.7. Ryan used invariably the visible corona for indication of initial breakdown; some of his results on wires of different size are plotted as circles in Fig. 2 where they are seen to be very irregularly located. Aside from the uncertainty of the method of observation, the wave form and frequency may have introduced considerable error in the results as reported, although that due to frequency would have tended to a lower rather than a higher value than for 60 cycles.

Further experiments on the variation of the pressure equation with the size of wire are in progress.

Influence of Temperature and Moisture.—No satisfactory investigation has been made of the influence of temperature on corona voltage. Ryan reports a series of observations on the visible corona for temperatures between 70° and 200° Fahrenheit. The size of wire is not stated. The results are admittedly wanting in accuracy, but indicate a linear relation between corona voltage and temperature; in fact, Ryan states that the maximum value of corona voltage varies inversely as the absolute temperature.

The writer has conducted a short series of tests between 6° and 41° C. on a .27-cm. wire for the purpose of obtaining a correction factor for his various observations as taken at different temperatures. The result as stated in the paper already referred to is that the relation is linear and that for each degree rise or fall from 21° C. there is a lowering or raising in the value of the critical voltage of 0.22 per cent.; Ryan's results indicate 0.27 per cent. for this value. Expressed in terms of surface intensity in kilovolts per centimeter and temperature in degrees Centigrade the writer's results may be expressed by the formula:

$$KV./Cm. = 61 - .132\,t. \tag{6}$$

In view of the observations of the effect of variation of pressure on different sizes of wire, it is not improbable that the constants of equation (6) will also vary with the size of wire. Further investigation in this direction is therefore desirable.

Moisture content up to amounts quite close to saturation have no effect on the values of voltage at which corona begins. While there is still some dissent from this opinion among electrical engineers, the author's results on this question, described in the earlier paper, appear very conclusive, and have been widely accepted. An influence of moisture on the amount of power loss above the critical voltage appears quite probable, in the light of the ionization theory in which the mass of the ionic carriers, which make up the current are an important factor in its value.

DISCUSSION.

So far as the question of the value of voltage at which corona will start on a given transmission line is concerned, it is probable that a solution will be reached sooner or later by means of experiments of the general character as those described above, supplemented by observations on existing lines. Also, there is good reason to suppose that a comparatively simple law will be found. For the surface intensity for any arrangement and size of cylindrical conductors, corresponding to a given voltage, may be expressed in terms of these constants; and the critical or corona intensity, under standard conditions of temperature and pressure, is a simple function of the diameter of the conductor. The relation between pressure within the range of the atmosphere, and critical voltage, for a given size wire, is linear; and although the slope of the linear relation changes with the size of wire there is good reason to suppose that a simple law connecting them can be found. Much the same may be said of the influence of temperature; preliminary experiments showing that the linear relation exists over a fairly wide range. The effect of stranding the conductor has been studied for only one size of strand as yet, but it seems a simple matter, with some further investigation, to express the effect of each of these influences in terms of the diameter of the conductor.

The influence of the frequency does not offer promise of expression as a simple relation; this influence is small however within the limits of frequency met in practice. The state of the atmosphere appears to be of small importance, for moisture does not influence the critical voltage, nor does its state as regards ionization, as is indicated by several considerations given in a later paragraph. Dirt and impurities which on settling cause irregularities on the surface of the wire, may lead to localized brush discharges; and if these are sufficient in number they may cause a noticeable loss below the normal critical voltage.

It is of great interest, however, to consider the results in their relation to present theories of the nature of the electric conductivity and breakdown of a gas. It is assumed that the reader is familiar with the general features of the theory of ionization. Under this theory the neutral atoms and molecules of matter may be separated into smaller charged particles, and the motion of these particles under electric force constitutes an electric current. In a gas there are always a small number of these free ions present; this number may be greatly augmented by Röntgen rays, ultra-violet light and other well known ionizing agents. When so ionized currents of magnitudes within easy measuring range are obtained between terminals subject to a difference of potential. If this difference of potential is increased, a point is reached where the current increases sharply, showing the presence of some new saurce of ionization. The theory states that these new ions are formed by the impact of those already existing, and moving with higher velocity in the increased electric field, with the neutral molecules of the gas. This phenomenon has been called ionization by collision or secondary ionization.

The results of the experiments which have been described above are for the most part consistent with the ionization theory. The various circumstances surrounding the appearance of corona all indicate that it is an instance of secondary ionization. Formula (1) indicates that near a conductor of large radius or near a plane, the corona intensity approaches a value 32 kilovolts per centimeter; secondary ionization between plane electrodes in closed vessels at atmospheric pressure has been noticed by several physicists to begin in the neighborhood of 30,000 volts per centimeter. The mass of elementary negative ion or electron is approximately 5.9×10^{-28} gms. and the charge it carries is 4.6×10^{-10} electrostatic units. In an electric field the mechanical force acting on the electron is the product of its charge and the strength of field. Hence by the laws of simple mechanics it is possible to calculate the acceleration, the velocity and the kinetic energy attained by an electron in moving a given distance under a given electric intensity. If the mean free path of the electron, about 6×10^{-5} cm. at atmospheric pressure, be the distance between collisions, it is thus easy to calculate the kinetic energy of the electron due to the electric field, when it collides with a molecule. This energy is readily seen to be equal to pVe, where p is the mean free path, V the electric intensity in electrostatic units, and e the charge of the electron. If now the voltage between plane parallel electrodes be raised until secondary ionization begins, the value of the voltage makes it possible to calculate the energy required to ionize a molecule of a gas. In fact the values of the energy required to ionize a molecule which are now generally accepted are largely based on determinations of the value of electric intensity at which secondary ionization begins. It has been pointed out above that the values of this intensity as determined by Townsend and others are in close agreement with the value 32,000 volts per centimeter indicated by equation (1) as the lowest value at which corona appears. To one skeptical as to the correctness of the theory of ionization therefore (and there are many such) all that may be said so far is that the phenomena of sudden increase of current above a certain value of electric intensity as observed by Townsend, and that of corona formation, are probably due to the same causes. But there are several other independent methods of determining the energy required to ionize a gas. The values are commonly expressed in terms of the potential difference in volts through which the electron must pass in order to acquire energy sufficient to produce an ion by collision. The value pertaining to the method described above is from 10 to 12 volts. Rutherford, from the relation between the heating effect of radium and the number of ions it produces, gives the value 24 volts. Stark and Langevin by independent methods conclude that the values are 45 and 60 volts respectively. While the extreme values differ by the factor 5 or 6 it must be remembered that the actual amount of energy required to produce an ion is about 5×10^{-11} ergs, so that all of these values indicate the same order of magnitude; therefore when taken together they constitute a very strong reason for supposing the value 5×10^{-11} ergs is close to the correct one. If this be true it is good evidence that the formation of the corona is actually due to the liberation of ions from the neutral molecules of the gas, when the latter suffer collision from a free electron moving under the force of the electric field. That the electron and not a gaseous ion or aggregate is the active agent is shown by the shorter free paths of these latter which by the relation already given results in a lower value of kinetic energy at the time of collision than those given above.

The writer has shown by stroboscopic methods that above the critical voltage the corona begins and ends at a point on the alternating current wave which corresponds very closely in every case with this critical value. It is well known that since secondary ionization depends only on the velocity of the ions and thus on the electric intensity, it should within wide limits be independent of the number of ions already existing in the gas. The corona stops sharply on the descending side of the voltage wave showing that the copious ionization present during the existence of corona does not aid it in persisting to a lower voltage than that at which it starts. The presence of a greater or less amount of free or spontaneous ionization in the atmosphere has been advanced by some writers to explain the discrepancies, among different observers, in the voltage at which the corona starts. The foregoing facts seem fairly conclusive that this supposition is not correct. In order, however, to further remove doubt on this point a simple experiment was performed in which the air surrounding the conductor was ionized from an independent source. A clean polished wire 15 cm. in diameter was stretched vertically along the axis of a cylinder 17.5 cm. in diameter and about 120 cm. long, made of woven wire with a 1 cm. mesh. The high voltage was applied between them, the wire cylinder being also connected to ground. A large Röntgen ray tube was enclosed in a light-tight box and placed close to the cylinder. When this tube was excited a crude electroscope placed 20 or 30 cm. on the other side of the cylinder was immediately discharged showing that the air of the neighborhood was strongly ionized. In the darkened room the starting of the visible corona on the wire could be located readily and the corresponding voltage determined by successive trials within an error of two or three tenths of one per cent. By the use of independent observers it was established without doubt that the presence of the Röntgen ray tube caused no variation in the value of voltage at which the corona starts.

The general influence of a decrease in pressure or an increase in temperature toward a lower critical voltage is quite consistent with the ionization theory. For under the kinetic theory of gases the free paths of the vibrating molecules and ions are lengthened in these two conditions. During the free path or interval between collisions the ions are acted on by the electric force, and the longer the interval the greater the velocity acquired and the more kinetic energy and ionizing power. Hence a given amount of energy will be acquired at a lower voltage if the free path is lengthened.

The lowering of the critical voltage by an increase in frequency is not to be explained so simply. However if within the molecule or atom there are a number of electrons in motion or free to move, and there is some indirect evidence to this effect, it is evident that the forced vibrations set up by an external alternating field will, with the increasing frequency of these vibrations, cause the mutual attractions within the structure of the atom to become less and less strong, and therefore more liable to be broken when in collision with an extraneous ion. It is surprising however that this effect should be noticeable at frequencies so low as 60 to 90 cycles, for they are incomparably slower than those suggested by theory for the vibrations within the atom. The close relation between the first appearance of corona and the peak or maximum of the voltage wave is natural in the light of theory, for at atmospheric pressure the mean free path of an electron is about 6×10^{-5} cm. long, and under a field sufficiently strong to ionize this path is traversed in about 2×10^{-12} seconds.

Perhaps the most interesting problem in connection with the phenomenon of corona formation is the explanation of the greater values of electric intensity required to start corona around smaller wires, *i. e.*, the upward trend of the curve of Fig. 2. Why should the properties of the air change with a slight alteration in the size of a conductor whose diameter is fifty thousand times as great as the mean free path of a molecule? No tenable explanation has been offered. The attraction to the conductor of oppositely charged ions which pile up as it were and reduce the actual gradient below that calculated, and at the same time increase the gas pressure, has been suggested. Both suppositions immediately include an influence on corona voltage of the amount of ionization already present, and this as already noticed is contrary to observation. Simple calculation also will show that the charge sufficient to materially reduce the gradient at the surface of a conductor at corona potential would require a number of ions far in excess of the numbers commonly present in the atmosphere. The writer by a sensitive optical method could find no indication of an increase of pressure at the surface of the conductor. It appears probable that the explanation will be found in the decreasing surface of the smaller conductors. Secondary ionization probably begins with the collisions of a few electrons which have free paths longer than the average. With decreasing area of conductor, the number of neighboring electrons whose free paths exceed a certain length, and at the same time are subject to the maximum electric intensity, will be decreased, and consequently the corona forming electric intensity must be higher.

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